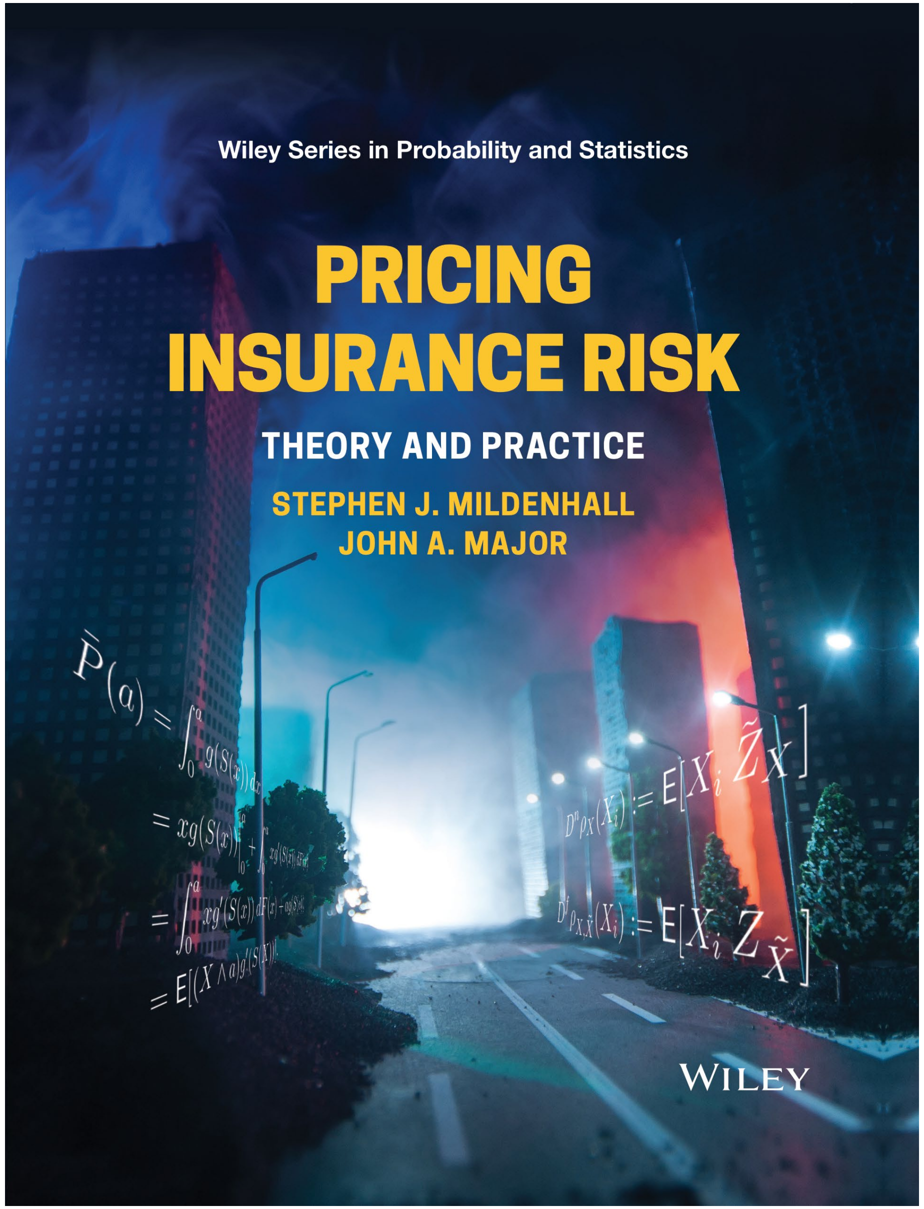
convex risk

Modern Pricing: Update your Methodology

Stephen J. Mildenhall

The Old Library, Lloyd's

January 17, 2023



Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Highlights

Adam Smith was ahead of his time. It takes two risk measures to price. The Cost of Capital (CoC) is not constant.

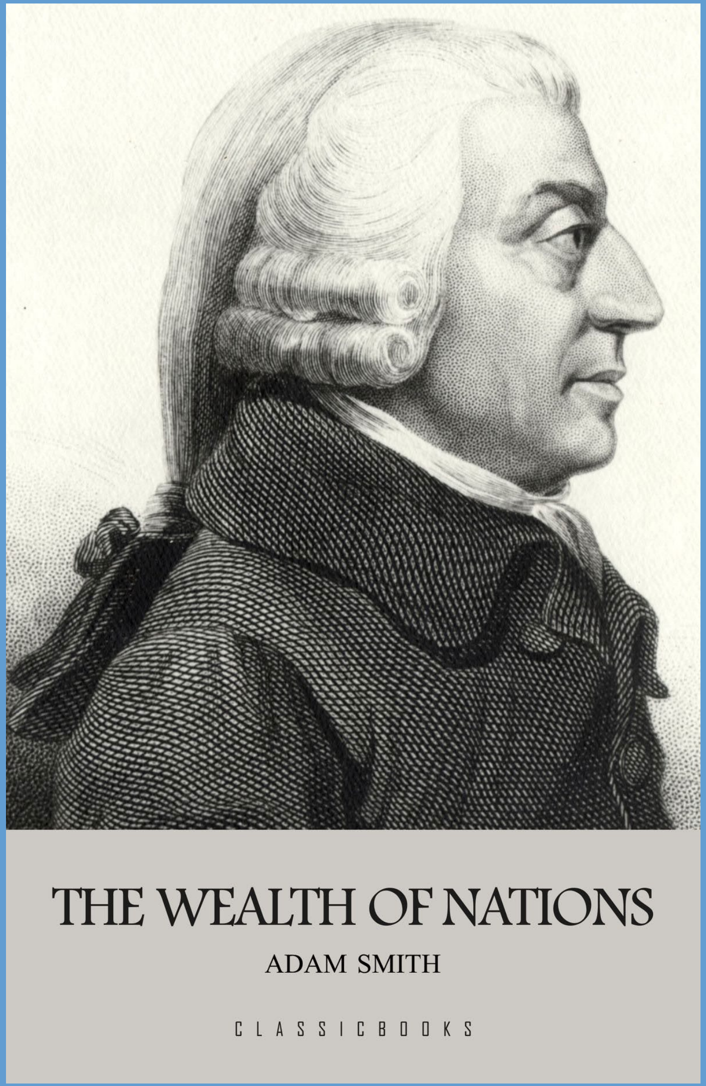
Modern finance provides a satisfying theoretical pricing model. But it doesn't quite work in practice and is hard to parameterize.

Allocate premium directly, not capital. Quantify risk-appetite unknowns. Impacts of distribution uncertainty.

Theory: The Answer

Theory: ~~The~~ Answer

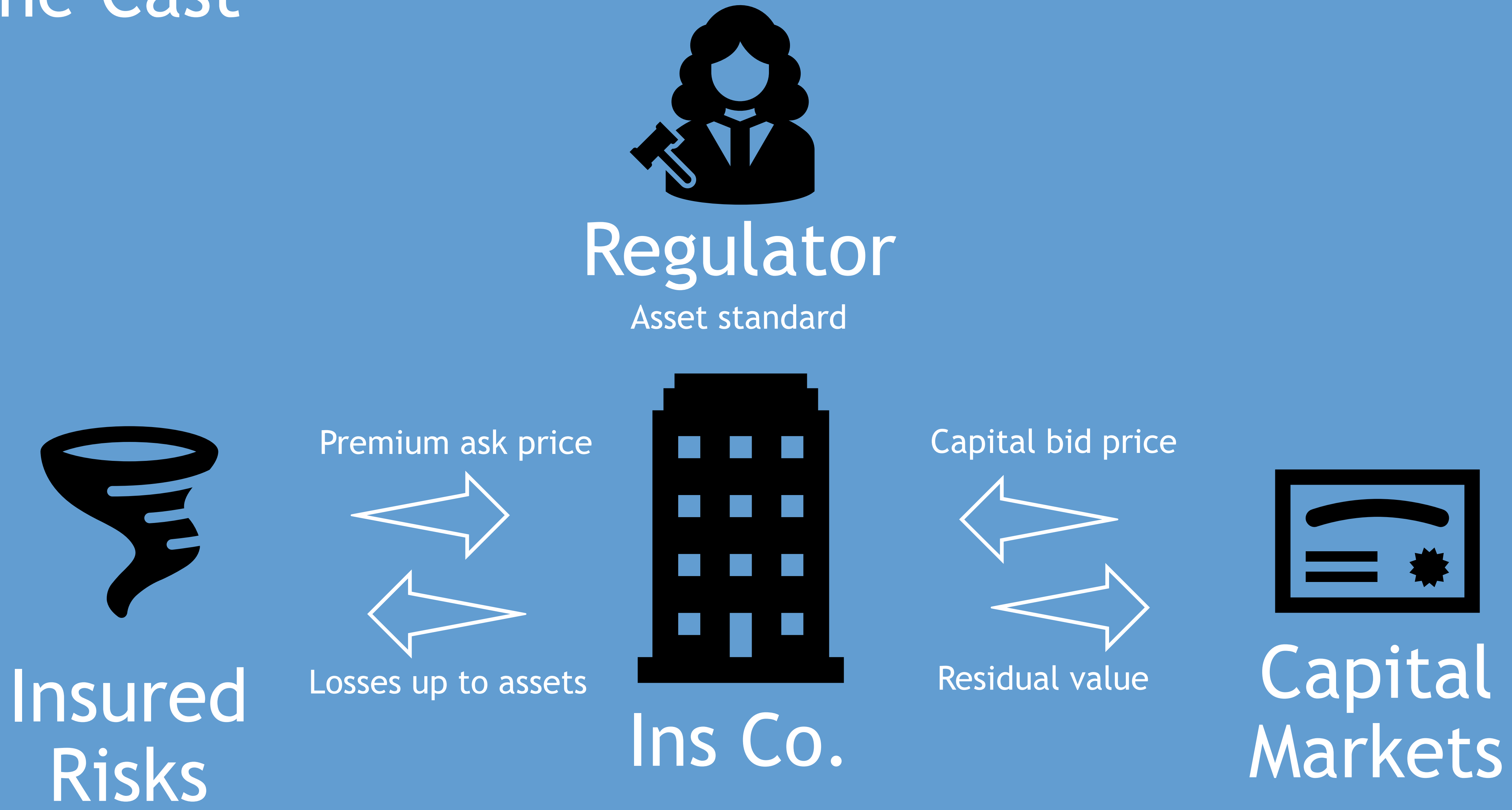
Theory: An Answer



In order to make insurance a trade at all, the common premium must be sufficient to compensate the common losses, to pay the expense of management, and to **afford such a profit as might have been drawn from an equal capital employed in any common trade.**

Book 1, Ch X, Part I, 5th Edition, 1789

The Cast


Regulator
Asset standard
Premium ask price
Losses up to assets
Capital bid price
Residual value
Insured Risks
Ins Co.
Capital Markets

Adam Smith Cost of Capital (CoC) **Portfolio** Pricing

Total loss X, plus two ingredients

- Amount of assets = a
- Cost of capital rate = ι

CoC premium formula

- P = (expected loss)
  + (cost of capital rate) x (amount)

  = EL + ι (a – P)

Observations

1. Assets = Premium + Capital
2. **Two** risk measures, a and ι
3. Asset amount a(X) is exogenous
4. Pricing $X \wedge a = \min(X, a)$, not X
5. Price related to capital structure

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CoC premium formula

- P = (expected loss)
  + (cost of capital rate) x (amount)
  = EL + ι (a – P)
  = EL + d (a – EL) — EL + risk margin
  = a – v (a – EL) — a – capital
  = v EL + d a — v E[·] + d max(·)

v = 1 / (1 + ι), risk discount factor

d = ι / (1 + ι) = ι v, rate of risk discount

- Method-in-use in US rate filings

Constant Cost of Capital? CoC *should* vary, but how?

"...the use of a company-wide cost of capital implicitly assumes that the new policy has the same risk-return characteristics as the firm as a whole. ...this assumption **may be questionable** in multiple line companies..."

Cummins, JRI 1990

- **CCoC across lines** originally resulted from estimation problems
- ...but extremely convenient: EVA, pricing becomes capital allocation
- Forgotten: credit yield curve shows CoC **varies across capital layers** (priorities)
- MM? Financing matters for allocation!

Classical and Modern Pricing: 1997

Shaun Wang, 1996
Premium calculation by transforming the layer premium density (ASTIN)
891 Google Scholar citations

Rich Phillips et al., 1998
Financial Pricing of Insurance in the Multiple-Line Insurance Company (JRI)
336 Google Scholar citations

Freddy Delbaen et al., 1997
Thinking Coherently (1997 RISK)
Coherent Measures of Risk (Math Fin)
11,928 Google Scholar citations

Modern Portfolio Pricing and Risk Measurement

Pricing functional properties

a) **Monotone**: Uniformly higher risk implies higher price

b) **Sub-additive**: diversification decreases price

c) **Comonotonic additive**: no credit when no diversification; if out-comes imply same event order, then prices add

d) **Law invariant**: Price depends only on the distribution; no cate-gorical "line" CoC

A **spectral risk measure** (SRM) ρ(X) is defined by properties (a)-(d). It has four representations:

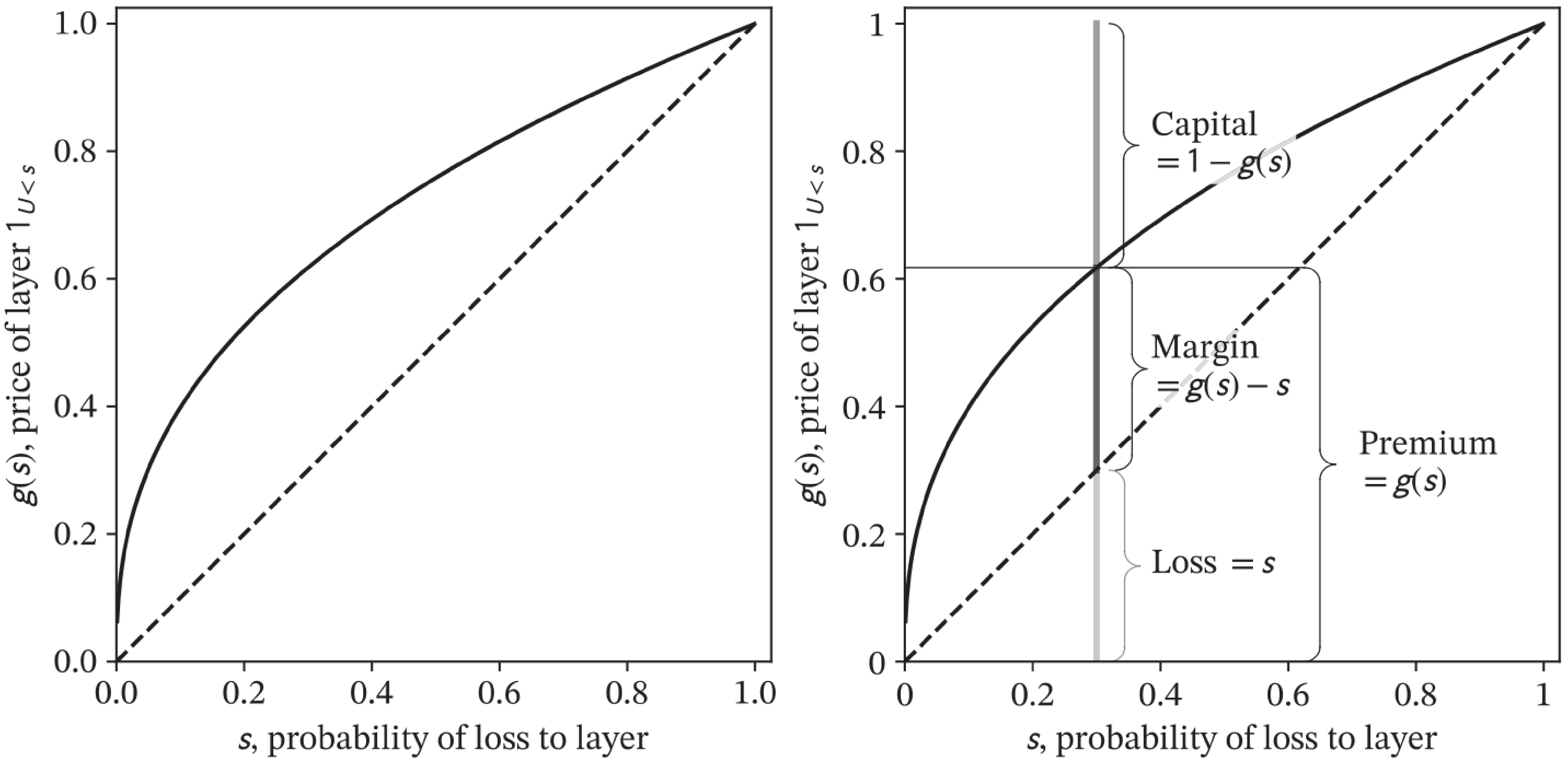
1. Cts ↑ weighted average of VaRs
2. Weighted average of TVaRs
3. Worst over a set of probability scenarios, max { E[XZ] | some Z }
4. Distorted expected value

$$\rho_g(X) := \int_0^\infty g(S_X(x))\,dx \quad = \mathsf{E}[Xg'(S(X))]$$

for increasing, concave g

- **Natural allocation**: $E[X_i\, g'(S(X))]$

Distortion Function: g(s) = Ask Price for Bernoulli 0/1 Risk



Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Spectral Risk Measure Portfolio Pricing

High asset layer: margin high % EL but low % capital; very low leverage

Low asset layer: margin low % EL but high % capital; very high leverage

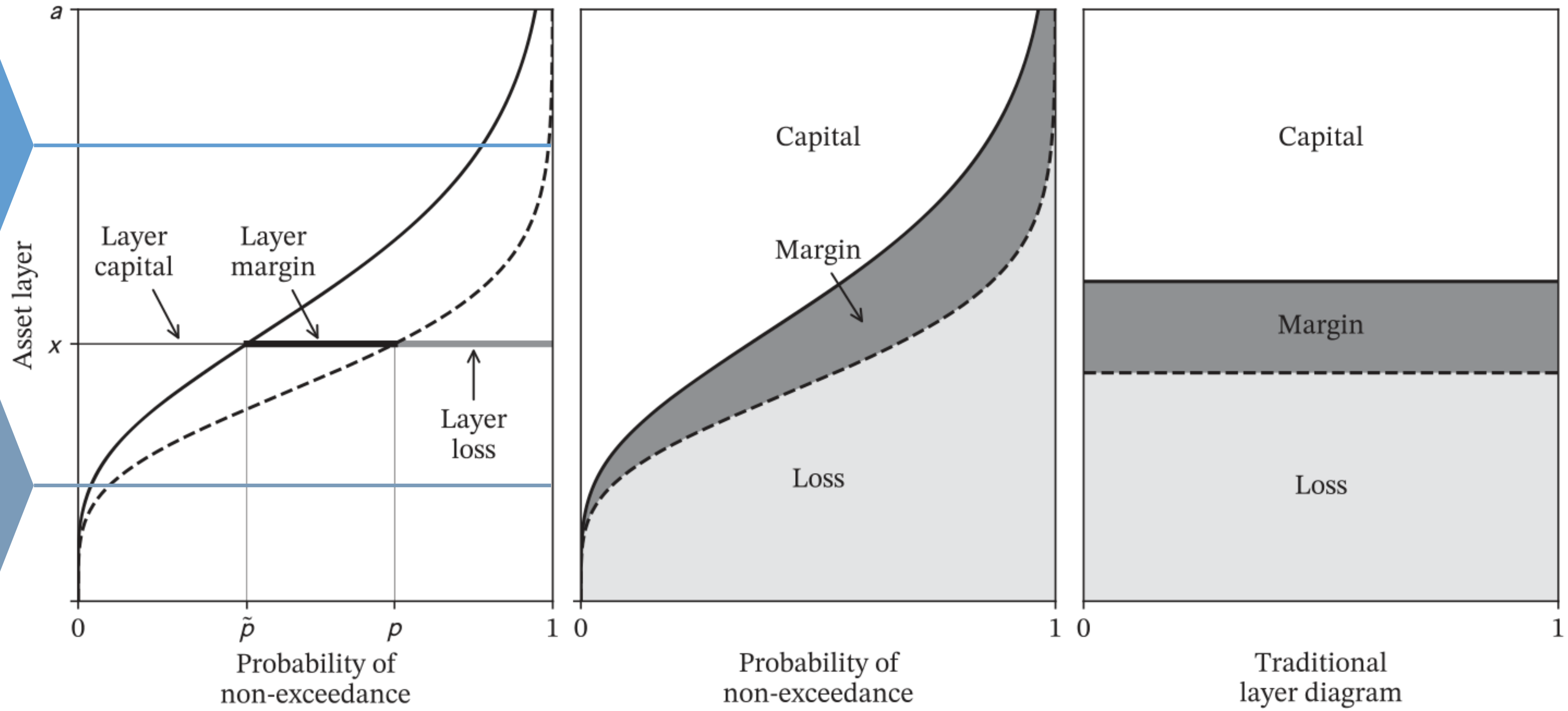


Figure 10.5 Continuum of risk sharing varying by layer of loss (dashed) and premium (solid): by layer (left), in total summed over layers (middle), and traditional (right). The total loss, margin, and capital areas are equal in the middle and right plots. Losses in the left and middle plots are Lee diagrams.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Premium Allocation: Marginal versus Natural

Marginal cost allocation

- Consistent with microeconomic optimization
- Euler or gradient allocation
- Cost based: **insurer's** perspective

- **C**CoC makes cost ↔ capital
- Tasche, EVA, Meyers, Myers-Read
- Venter/Major/Kreps (ASTIN, 2006): directional derivative

Natural allocation $E[X_i\, g'(S(X))]$

- Intuitive and consistent with modern finance, risk adjusted probabilities
- Risk adjustment = $g'(S(X))$
- Benefit based: **insured's** perspective

- Venter/Major/Kreps: decomposition and co-measure
- Must know payments in default

...cannot always be equal

Delbaen's Theorem (2000): Marginal Equals Natural...

If there is a **unique** density Z so that ρ(X) = E[XZ], then the marginal allocation equals the natural allocation

Comments

- Always have Z = g'(S(X))
- Qu: Are there others?
  Ans: No, iff q_X is increasing
- When Z not unique, choices include
  - **Linear** natural: Z ← E[Z | X] (lower left)
  - **Lifted** natural: use X for X ∧ a (lower right

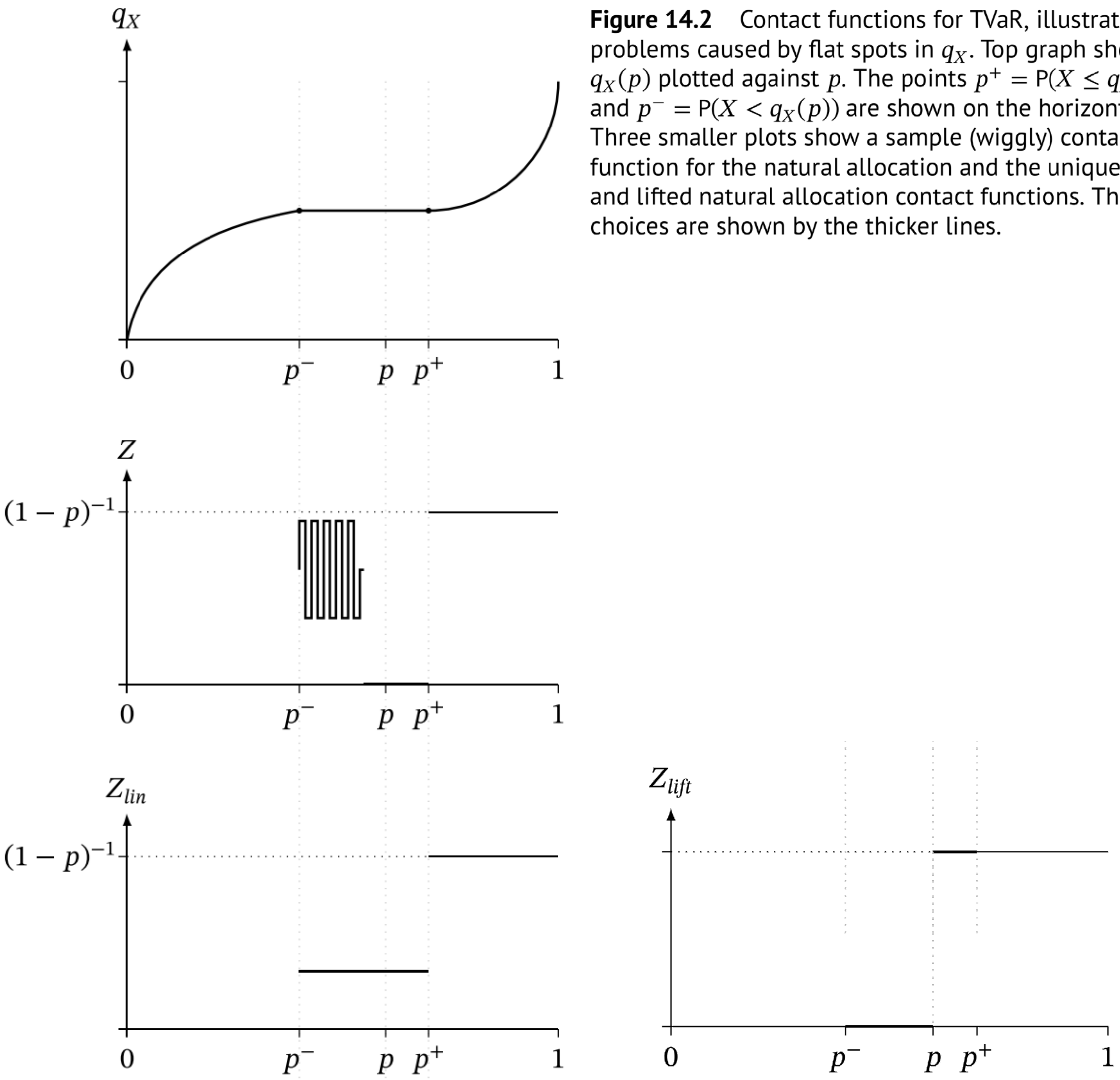
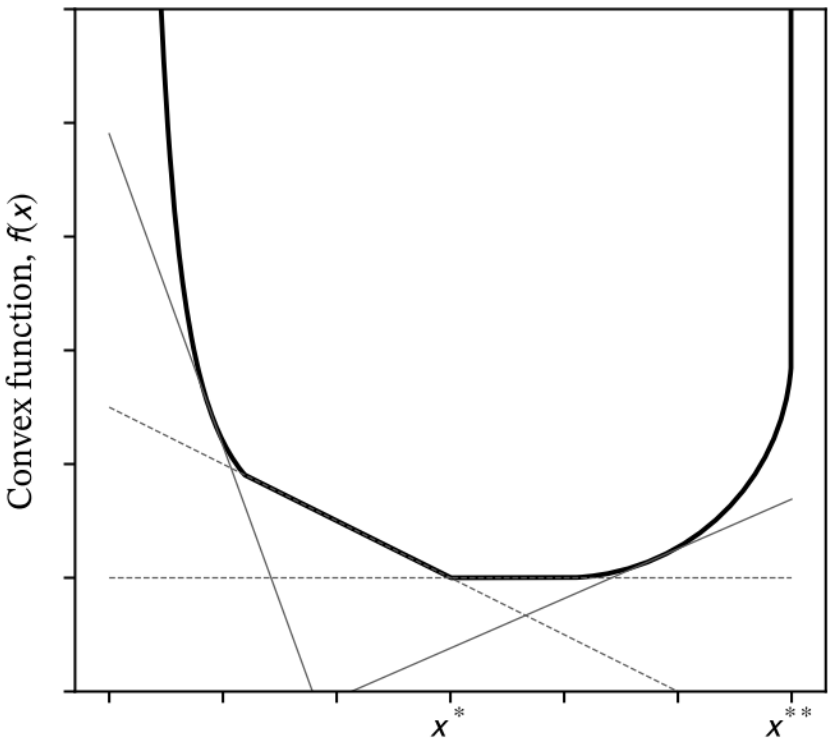


Figure 14.2 Contact functions for TVaR, illustrating the problems caused by flat spots in q_X. Top graph shows $q_X(p)$ plotted against p. The points $p^+ = \mathsf{P}(X \le q_X(p))$ and $p^- = \mathsf{P}(X < q_X(p))$ are shown on the horizontal axis. Three smaller plots show a sample (wiggly) contact function for the natural allocation and the unique linear and lifted natural allocation contact functions. The choices are shown by the thicker lines.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

A Beautiful Theory

- Marginal = Natural allocation unless
  - Convex ρ is not differentiable at X
  - Marginal left/right derivatives different
  - The order of writing matters
  - Default rules matter
  - **Working with $X \wedge a$ and $\Pr(X>a) > 0$**
- Delbaen: best answer to Venter, Major, Kreps
- Applies to homogeneous and inhomogeneous portfolios



Modern finance provides a satisfying theoretical pricing model. But it doesn't quite work in practice.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley

Practice: The Allocations

To Apply SRMs Must Solve Two Problems…

1. Non-uniqueness caused by the flat spot in $X \wedge a$

- No definitive answer
- Linear and lifted allocations

2. Determine distortion function g

a) Directly

b) Indirectly

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2a) Estimating g Directly

- Interpretation: g(s) is the ask price to write the Bernoulli risk with probability s of loss (s small)

- Comparables
  - Corporate bonds
  - Catastrophe bonds (perfect example Ins Co.!)
  - No data for s > 0.2

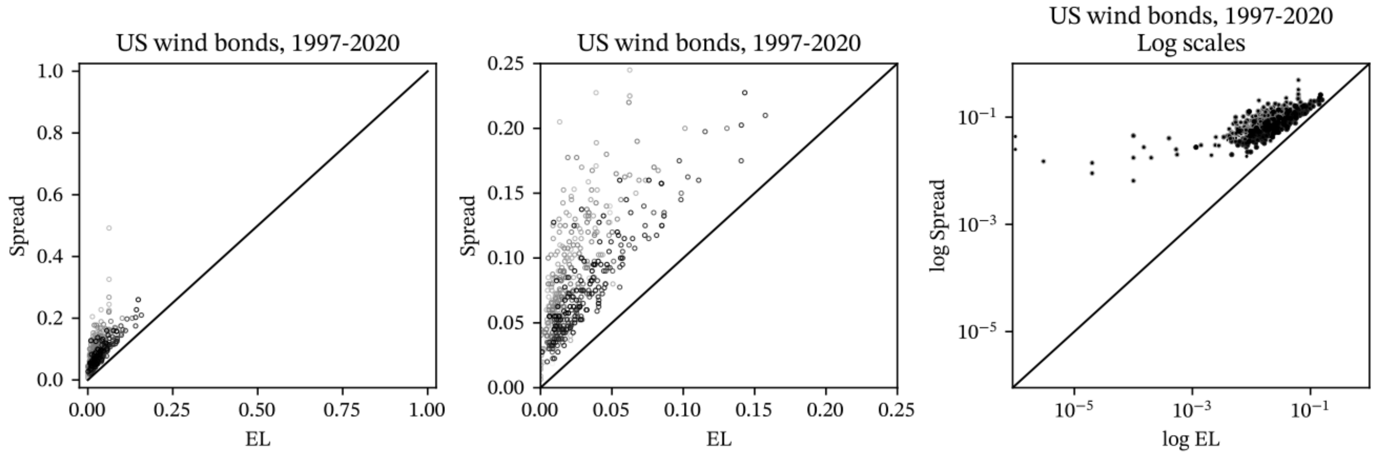
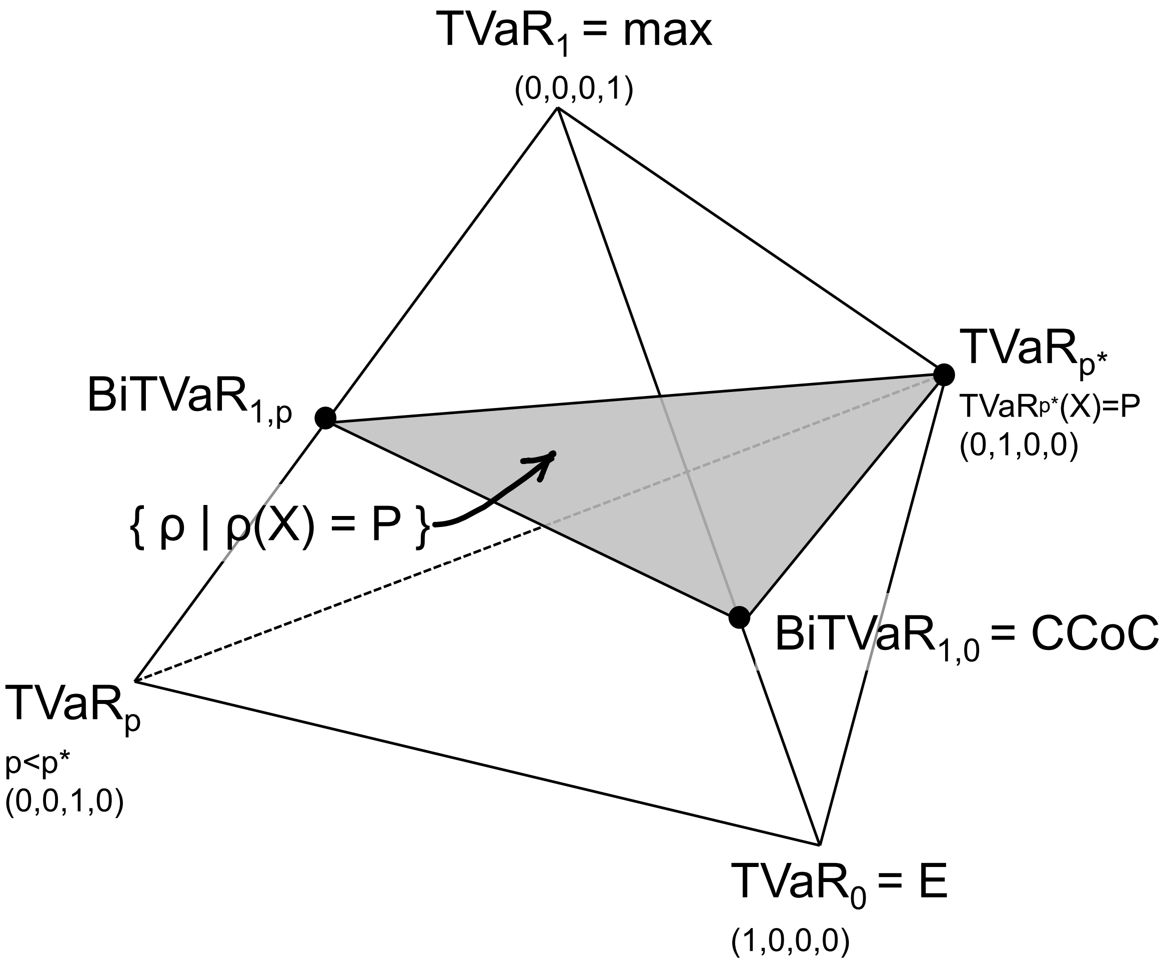
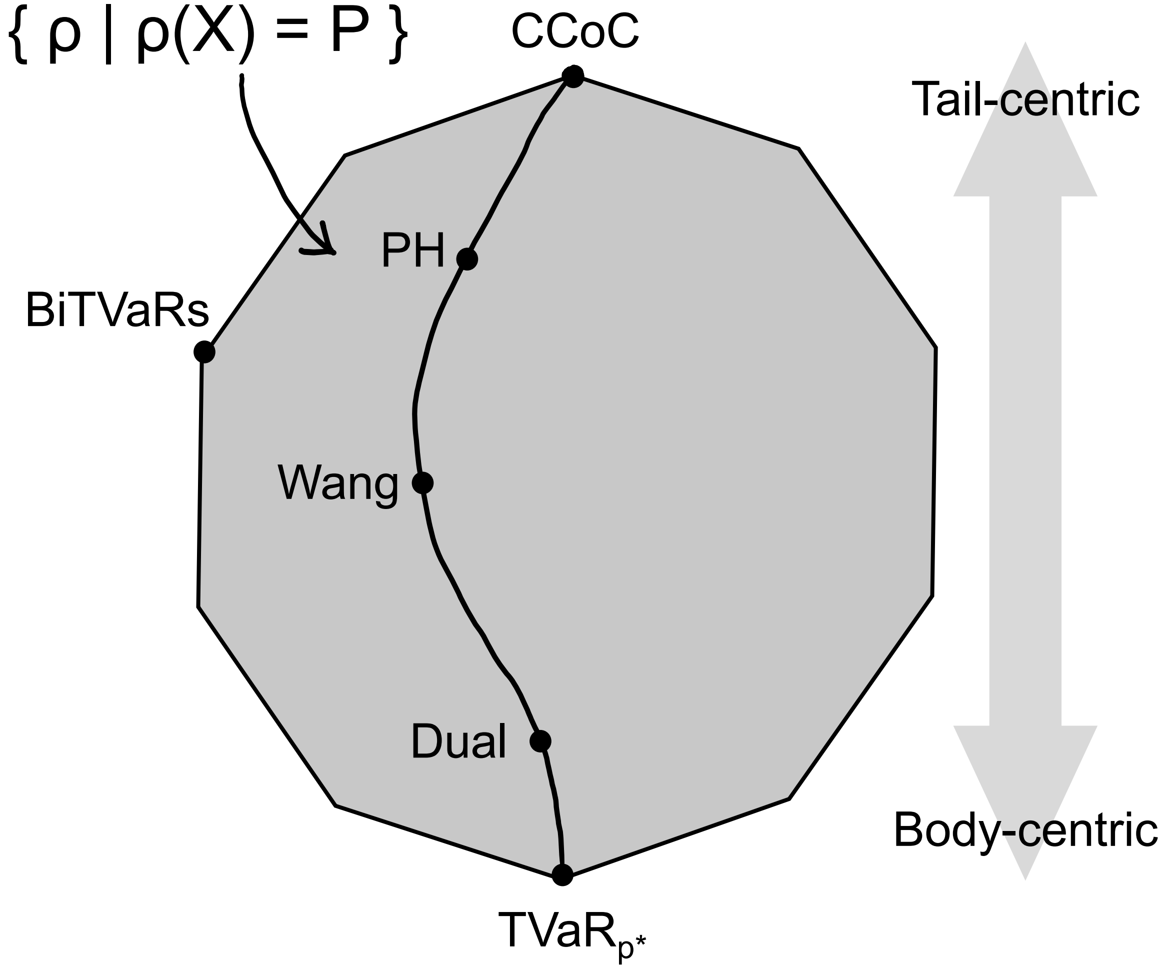


Figure 11.12 Spread (ROL) vs. EL on US wind (hurricane)-exposed catastrophe bonds since 1996. More recent years are shown darker black. The left and middle plots differ only in scale. Notice that catastrophe bond data includes observations for only $s < 0.20$. The right hand plot is on a log scale, emphasizing highly rated (low default probability) bonds, and illustrating the well-known minimum-rate-on line phenomenon of reinsurance pricing. Data: Lane Financial LLC.

Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley; Data Lane Financial, LLC

2b) Inferring g Indirectly: Determine All ρ With ρ(X) = P





Mildenhall – Similar risks have similar prices, IME 105 (2022): 203-210.

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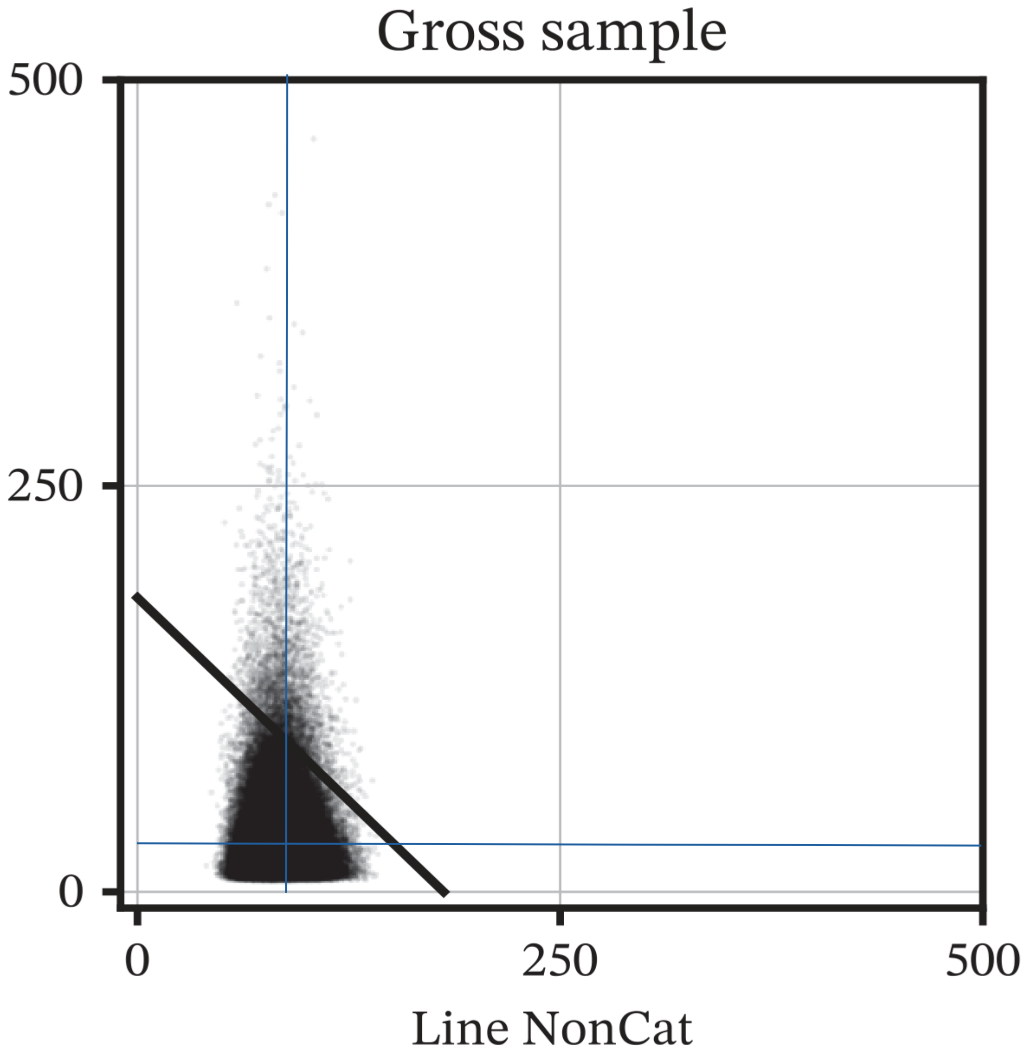
Cat/NonCat Case Study

Stochastic Model

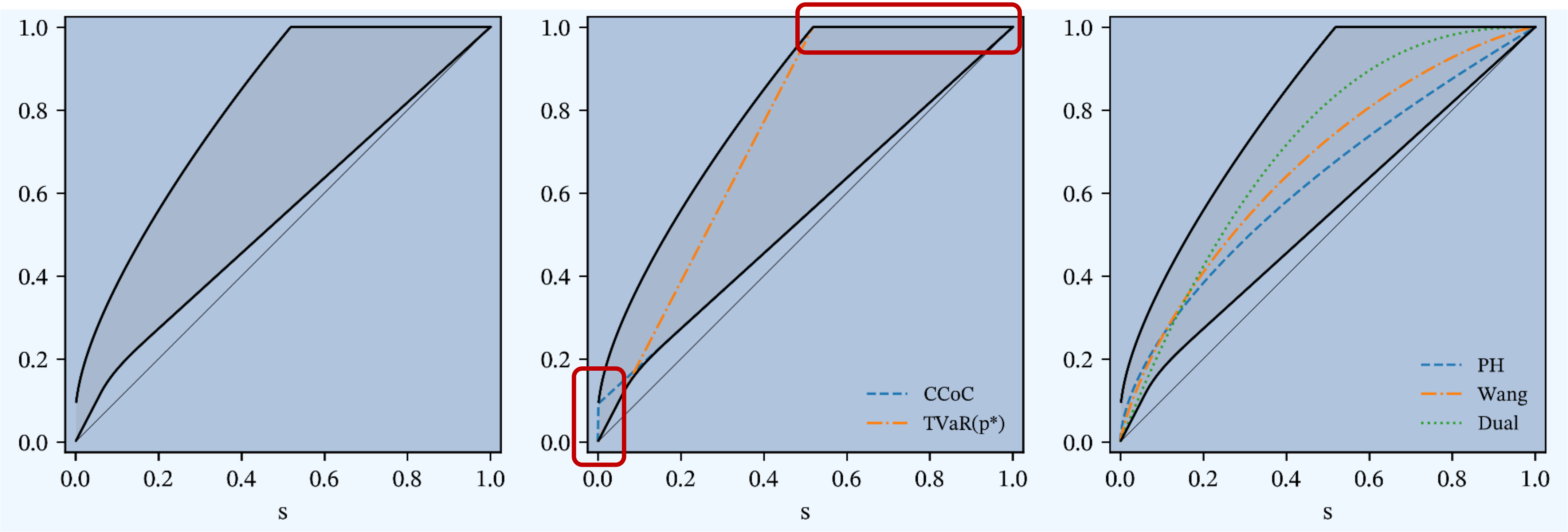
- NonCat: gamma, mean 80, CV 0.15
- Cat: lognormal, mean 20, CV 1.0
- Independent
- Total: mean 100, CV 0.233

- 99.9% VaR asset requirement, 267.2
- Calibrate pricing to 115.15, the 10% CCoC pricing
- Links at the end for Python code and complete set of Case Study exhibits



Graphic: Pricing Insurance Risk, Mildenhall & Major (2022), Wiley; Data Lane Financial, LLC

Distortion Envelope and Inferences about Net Risk



- Any distortion pricing gross Cat/Non-Cat to P=115.15, lies in shaded region
- CCoC **tail-centric**: greatest as s → 0, most expensive tail capital
- $TVaR_{p^*}$ **body-centric**: greatest near 1, cheapest tail, most expensive body capital

Distortion Envelope and Inferences about Net Risk

It is easy to compute maximum and minimum net prices consistent with given gross prices, bounding rational reinsurance "walk-away" price

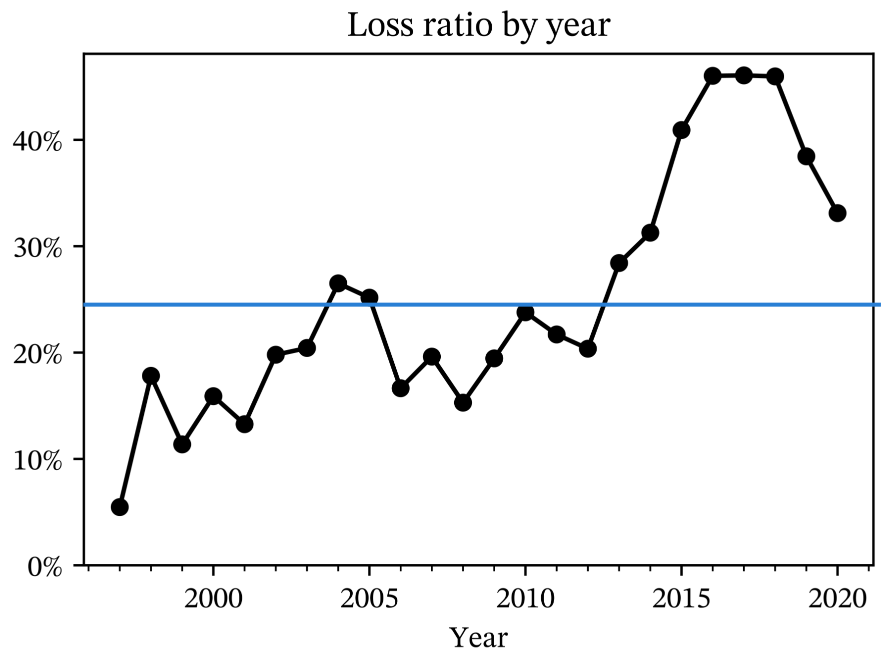
Application: Determine maximum economic spend for aggregate reinsurance 80 x 41 on the cat line (attachment probability 10%, detachment 0.5%)

- Net premium acceptable to investors ranges between
  - 105.90 for ρ = CCoC, most tail-centric (mgmt. concerned with "tail vol") and
  - 110.88 for $\rho = \mathrm{TVaR}_{p^*}$, most body-centric ("earnings vol")
- Implied maximum budget for reinsurance ranges between
  - 4.27 (115.15 – 110.88 for TVaR_{p^*}) and
  - 9.25 (115.15 – 105.90 for CCoC)

Distortion Envelope and Inferences about Net Risk

Revealed reinsurance budget between 4.27 and 9.25 depending on investor risk appetite (tail vs. body risk sensitivity)

- Ceded loss from program: 2.22
- Walk-away (lowest acceptable) ceded LR
  - 24% for CCoC-risk appetite: buy
  - 52% for TVaR: price sensitive
- **Range of outcomes brackets typical cat pricing: risk appetite material to decisions!**



Average loss ratio by year 1997-2020, US wind exposed bonds only.
Data: Lane Financial LLC.

Known Unknowns

A physicist, an engineer, and an economist…

Image: MidJourney, from prompt “three hungry people on a desert island one with large spectacles, background shows coconut trees and sea, standing next to a giant tin can”


Climate change
Inflation
Emerging risks
...cyber
Reserves

convex risk

Climate Change: Impact on Cat Line

Projections of Changes in U.S. Hurricane Damage Due to Projected Changes in Hurricane Frequencies

Stephen Jewson[a]

[a]Lambda Climate Research, London, UK

Hurricane Category	2°C Mean Hurricane Freq. Change	2°C SD Hurricane Freq. Change
1	1.011	0.3179
2	1.095	0.4176
3	1.134	0.4638
4	1.179	0.5174
5	1.236	0.5830

Table 2 *Means and standard deviations of the distributions of hurricane frequency adjustments we apply for a 2°C climate change scenario. A value of 1 is no change, and 1.1 is a 10% increase in frequency. The distributions of frequency change are log-normal, with the given mean and standard deviations. The distributions for different storm categories are perfectly correlated.*

Increase mean

- More events
- 15% increase

Increase volatility

- Uncertainty 44% CV

Adjust Cat line X_c to

$$\mathbf{1.15 \times U \times X_c}$$

U ~ lognormal, mean 1 and CV 0.44

Projections of Changes in U.S. Hurricane Damage Due to Projected Changes in Hurricane Frequencies (2022), under peer review

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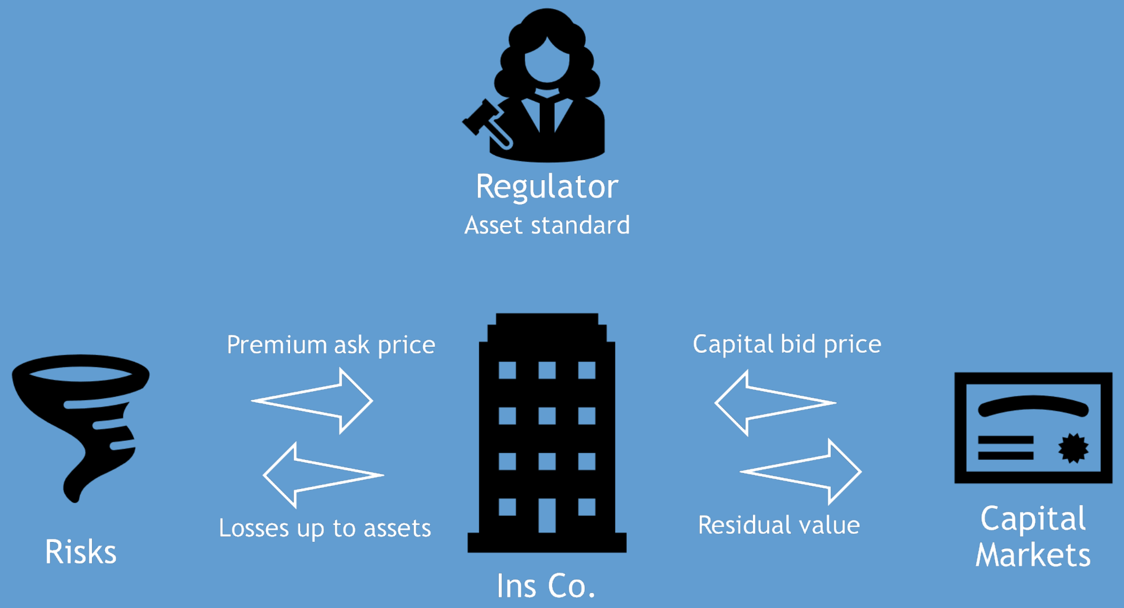
Climate Change: Impact on Portfolio and Its Economics

	Base Amt	Mean Amt	Mean Chg	Vol Amt	Vol Chg
Gross EL	99.9	102.9	3%	102.9	3%
Net EL	97.7	100.4	3%	99.8	2%
Ceded EL	2.2	2.5	15%	3.1	41%
Gross Assets	267.2	294.7	10%	347.1	30%
Reins Limit	79.6	91.6	15%	115.3	45%
Net Assets	187.6	203.1	8%	231.7	24%
Gross Prem	115.2	119.6	4%	121.0	5%
Net Prem	110.1	113.8	3%	113.8	3%
Reins Budget	5.0	5.8	16%	7.2	44%
Reins LR	44%	44%	-0%	43%	-1%

Notes
- Mean columns: 15% increase effect only
- Vol columns: mean plus 44% CV uncertainty
- Reins has same attachment/detach-ment probabilities
- Dual distortion
- Calculations in aggregate

Wrap-Up


Regulator
Asset standard
Premium ask price
Capital bid price
Risks
Losses up to assets
Residual value
Capital Markets
Ins Co.

Try for Yourself, Back at the Office…

- **Objective**: Create prospective by-unit planning benchmarks / technical premiums / reservation prices
- **Inputs**:
  - By-unit multivariate ultimate underwriting outcome distribution (sample)
  - Operating plan that supports target valuation, regulatory capital constraint

Try for Yourself, Back at the Office…

- **Objective**: Create prospective by-unit planning benchmarks / technical premiums / reservation prices

- **Inputs**:
  - By-unit multivariate **ultimate** underwriting outcome distribution (sample)
  - Operating plan that supports target valuation, regulatory capital constraint

- **Calculations**:
  1. Determine target total underwriting income as pre-tax plan income less anticipated investment income less other income
  2. Calibrate SRM distortion *g*-functions to target total underwriting income
  3. Linear natural allocation premium by-unit determines target combined ratios

  Allocate premium directly, not capital. Details in Pricing Insurance Risk book!

Conclusions

1. SRMs are a practical pricing tool

3. Parameterize to total premium and quantify range of appetite-consistent by-unit technical premiums

2. Firm's g encodes investor's view of bus-iness and management; encodes risk appetite

4. Three material axes of uncertainty: risk appetite, expected loss, volatility

Contact Information and Resources

Contact

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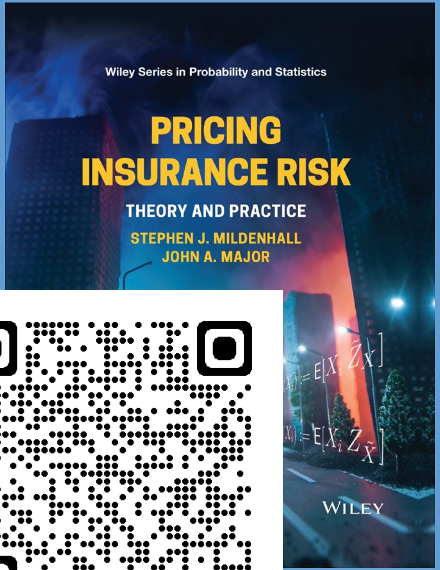
Book website



pricinginsurancerisk.com

- Case study exhibits
- Supplemental exhibits
- Presentations
- Errata

Cat / NonCat

Exhibits

Colab workbook

`aggregate` software

Documentation

Source

https://www.github.com/mynl.aggregate
https://aggregate.readthedocs.io/en/latest/